Mathematical Induction 3

1. Prove that : $\frac{d^n}{dx^n}\sin x = sin\left(x + \frac{n\pi}{2}\right)$

2. Prove $1 \times 3 + 3 \times 5 + 5 \times 7 + \cdots + (2n-1)(2n+1) = \frac{n}{3}(4n^2 + 6n - 1)$

   by mathematical induction.

3. Prove $1^2 + 2^2 + 3^2 + \cdots + (2n)^2 = \frac{n(2n+1)(4n+1)}{3}$ by any method.

4. $\int_0^{2\pi} sin^{2n} x dx = \frac{2\pi(2n)!}{(n!)^2(4^n)}$

5. Prove:
   $(1^5 + 2^5 + \cdots + n^5) + (1^7 + 2^7 + \cdots + n^7) = 2(1 + 2 + \cdots + n)^4$.

6. Prove $5^{2n+1} + 11^{2n+1} + 17^{2n+1}$ is divisible by 33 for all non-negative integer value of n.

7. Prove, by Mathematical Induction, that $n(n+1)(n+2)(n+3) \ldots (n+r-1)$ is divisible by $r!$ for all natural numbers n, where $r = 1,2, \ldots$